Assessing Topological Consistency for Collapse Operation in Generalization of Spatial Databases

Hae-Kyong Kang[1] and Ki-Joune Li[2]

[1] Department of GIS,
[2] Department of Computer Science and Engineering,
Pusan National University, Pusan 609-735, South Korea
{hkang, lik}@pnu.edu

Abstract. Generalization of spatial databases consists of complicated operations including not only geometric transformations but also topological changes. The changes often result in an inconsistency between the original and derived databases. In order to control the quality of derived databases, we must assess the topological consistency. In this paper, we propose a set of rules to assess topological consistency for the collapse operation. The rules are based on a rigorous classification of topological properties in a collapse operation. By these rules, we can detect inconsistent topological changes in the generalization process and improve the quality of derived databases.

1 Introduction

Multi-scale database is a set of spatial databases on the same area with different scales. In general, databases with small scales can be derived from a large scale database. We call this derivation procedure *generalization of spatial database*. It contains a number of complicated operations such as aggregation, collapse, simplification, and translation, which result in a considerable transformation from the source database.

In the generalization process of spatial databases, a problem arises from the inconsistency between the source and derived databases. It results in a quality degradation of the derived database. In order to control the quality of the generalization process, the consistency between the source and derived databases must be maintained. In this paper, we propose a method to assess the topological consistency of derived database.

During the generalization process, not only geometries but also topologies in the source database are to be changed. While most of topological relations in the source database should be maintained in the derived database, a part of topologies in the source database cannot be identical with those in the source database.

For example, figure 1 shows topological changes from a source database SDB to three other derived databases $MSDB_1$, $MSDB_2$, and $MSDB_3$. Two spatial objects A and B are polygons in the source database SDB. On the other hand, in $MSDB_1$ the two objects ar a polygon and line. In $MSDB_2$, they are simply

J. Akoka et al. (Eds.): ER Workshops 2005, LNCS 3770, pp. 249–258, 2005.

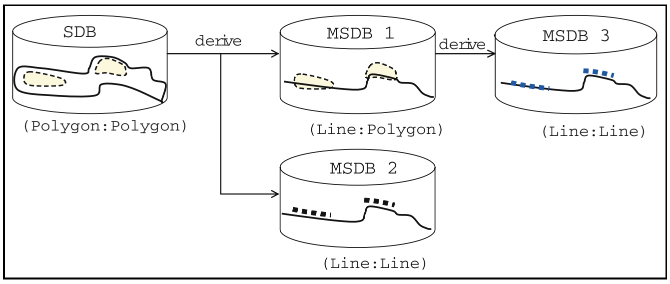


Fig. 1. Topological Changes in Generalization

lines. Figure 1 shows the change of topology from polygon-polygon topology in SDB to line-line topology in $MSDB_2$ and $MSDB_3$. This indicates that the topologies in derived databases can differ from those in the source database. In spite of the difference of topology, a set of correspondences are found between the topologies in source and derived databases. If the topologies in a derived database does not respect the correspondence, it implies that the derived database is not topologically consistent with the source database. Consequently, we can control the topological quality of derived databases by the correspondences.

Topological consistency issues are important in quality control of derived databases on multi-scale databases. Nevertheless, little attention has been paid on the issue. It is significant, however, to reconize such efforts as [6, 13, 15]. Specifically, Tryfona and Egenghofer' research[13] have made a pionering headway for the *aggregate* operation in the field. Now, we intent to continue where they left off.

The goal of this paper is to discover the correspondences between topologies of source and derived databases in case of the *collapse* operation rather than the *aggregate* operation. We propose a classification method of collapse operation by a boundary-interior model. A set of correspondence rules are proposed based on this classification. Such describe consistent topological changes to derived databases.

We discuss related work to our study in section 2. In the next section, we clarify the requirements for assessing topological consistency. Thus proposing correspondence rules to maintain topological consistency in derived databases. Finally we conclude the paper in section 5.

2 Related Work and Motivation

Topological relationships in source database are transformed to different but consistent ones on multi-scale databases. In this case, similarity or consistency between transformed relations and its original relations need to be evaluated. In [6], a boundary-boundary intersection was proposed to assess similarity of two relations on multi-scale representations. The boundary-boundary intersection is part of 9-intersection model [5]. If boundary-boundary intersections of two relations are same each other, the two relations are considered as same. The idea was developed based on the monotonicity assumption of a generalization; any topological relations between objects must stay the same through consecutive representations or continuously decrease in complexity and detail. In [13], a systematic model was proposed to keep constraints that must be held with respect to other spatial objects when two objects are aggregated. This work can be a solution when a multi-scale database is derived by aggregation. However, a solution is still lacking for multi-scale databases derived by a collapse operation from a source database.

The 9-intersection model[2, 5] is used as the topology model of this paper to represent relationships between spatial objects. According to the researches, 8 topological relations(`disjoint`, `contains`, `inside`, `equal`, `meet`, `covers`, `covere-`, `dBy`, `overlap`) between a polygon and a polygon, 19 topological relations($PL_1 \sim PL_{19}$) between a polygon and a line, and 33 topological relations($LL_1 \sim LL_{33}$) between a line and a line are defined. These relations will be referred to throughout this paper.

3 Classification of Collapse Conditions

In this section, we present, with an example, the requirements for assessing and maintaining topological consistency in collapse operation. The collapse operation will be classified according to the topology between the original and collapsed objects.

In figure 2, two polygonal objects A and B in the source database have MEET topological relationship according to the definitions proposed by [2]. Suppose A and B are collapsed to a and b in a derived database respectively. Accordingly, we see that the topology of derived database depends on the type of collapse operation. As depicted in figure 2(a), the topology between a and b may be MEET as the source database, while the topology between a and b in figure 2(b) is not MEET but DISJOINT. This difference comes from different topological relationships between A and a. In figure 2(a), $a \subset A^\circ$ while $a \cap A^\circ \neq \varnothing$ and $a \cap \partial A \neq \varnothing$ in figure 2(b), where ∂A and A° mean the boundary and interior of A respectively.

From this example, we observe that the topological relationship between the original object and collapsed object should be carefully examined as well as topological relationships between two objects in source database. The topology between the original object and collapsed object is determined the type of

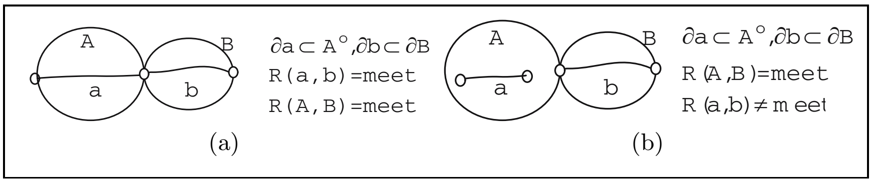
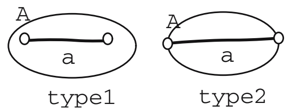
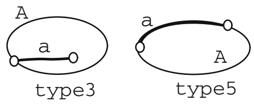
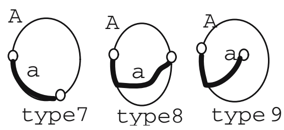


Fig. 2. Example of topological changes

collapse condition, which can be described by topology between boundary and interior of the source and collapsed objects. While we assume that the collapsed object is always contained or covered by the original object, the topology of exterior does not need to be considered.

The topologies between the original and collapsed objects are classified as table 1. Due to the paper length constraints, we assume that the geometric type of collapsed object is line. It may be important in real applications, but we can extend the method proposed in this paper to handle point objects with ease. In the table 1, although there are 9 types of collapse operations, collapse type 4 and 6 do not exist in practice, due to the continuity of line object.

Table 1. Collapse Types

R(A,a)		example
Collapse-Type #	Derivation of A_C from A	
type 1	$a^\circ \subset A^\circ$ $\partial a \subset A^\circ$	
type 2	$a^\circ \subset A^\circ$ $\partial a \subset \partial A$	
type 3	$a^\circ \subset A^\circ$ $\partial a \cap A^\circ \neq \varnothing \wedge \partial a \cap \partial A \neq \varnothing$	 (collapse-type 4 and 6 are impossible because of the continuity of line)
type 4	$a^\circ \subset \partial A$ $\partial a \subset A^\circ$	
type 5	$a^\circ \subset \partial A$ $\partial a \subset \partial A$	
type 6	$a^\circ \subset \partial A$ $\partial a \cap A^\circ \neq \varnothing \wedge \partial a \cap \partial A \neq \varnothing$	
type 7	$a^\circ \cap A^\circ \neq \varnothing \wedge a^\circ \cap \partial A \neq \varnothing$ $\partial a \subset A^\circ$	
type 8	$a^\circ \cap A^\circ \neq \varnothing \wedge a^\circ \cap \partial A \neq \varnothing$ $\partial a \subset \partial A$	
type 9	$a^\circ \cap A^\circ \neq \varnothing \wedge a^\circ \cap \partial A \neq \varnothing$ $\partial a \cap A^\circ \neq \varnothing \wedge \partial a \cap \partial A \neq \varnothing$	

Based on the classification of collapse conditions, we can derive a set of rules to describe the consistent correspondence between topologies of the original and derived databases.

4 Rules for Accessing Topological Consistency

The goal of this study is to define a set of rules for assessing the topological consistency between the original and derived databases in case of collapse operation. In other words, we should find the correspondence rules between the

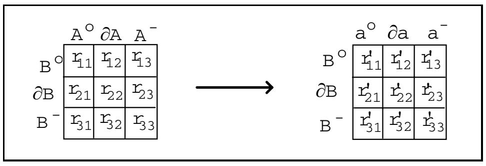


Fig. 3. Correspondence of Topological Relationship

9-IM matrix of the original and derived databases, as depicted by figure 3. If the 9-IM matrix in the derived database would differ from the matrix derived by the corresponding rules, then that the derived database is not topologically consistent with the original database.

In this section, we therefore propose a set of rules for finding the the corresponding matrix of the derived database under given topological conditions in the source database. They are based on the classification of collapse types given in section 3. The form of rules is as $Rule : R(A, B), R(a, A) \Rightarrow R(a, B),$ where a is the collapsed object from A, $R(A, B)$, $R(a, A)$, and $R(a, B)$ are the topological relationships between A and B, a and A, and a and B. Note that the topological relationship $R(a, A)$ is determined according to collapse types defined in section 3.

The topology between two objects A and B in the source database may be described by 8-topology model, or 9-Intersection Model [2]. However, here it is classified into five cases as follows, according to point set expression. More detail classification of topology in the original database may be possible, but it leads to the same result.

- case 1 (equal set) :$A = B$
- case 2 (subset) : $A \subset B$
- case 3 (superset) : $A \supset B$
- case 4 (disjoint) : $A \cap B = \varnothing$
- case 5 (intersect) : $A \cap B \neq A, B, \; A \cap B \neq \varnothing$

Based on the five cases and the classification of collapse types defined in section 3, we derive 16 rules as summarized in table 2. For the purpose of succinctness, we omit the proof of these rules in this paper. Table 3 and 4 explain how to apply them for assessing topological consistency. Such can be clearly illustrated with an example.

Suppose $A = B$ and the collapse type is `type 1`. In order to find the topological relationship between the interior of the collapsed object a (a°) from A and B, we apply `rule 1` in table 3. Similarly, we apply `rule 3` and `6` to find the topological relationship between the boundary of a (∂a) and B for the case where $A \subset B$ and the collapse type is `type 7`.

The way to apply the rules is summarized by table 4. Note that due to the limit of paper length, it is explained only for the case where $A = B$. We can easily extend this table for the rest cases. For example, suppose that the collapse type is `type 1` and $A = B$. Then we can derive $R(a, B)$ as follows,

Table 2. Assessing Rules for Topological Consistency

Category R(A,B)	Assessing Rule: rule #	Assessing Rule: definition
Equal-set Rule (A=B)	rule 1	$P_o^a \subset P_i^A,\ P_l^B = P_i^A \Rightarrow$ $P_o^a \cap P_l^B \neq \varnothing,\ P_o^a \cap P_m^B = \varnothing,\ P_o^a \cap P_n^B = \varnothing$
	rule 2	$P_o^a \cap P_i^A \neq \varnothing,\ P_o^a \cap P_j^A \neq \varnothing,\ P_l^B = P_i^A,\ P_m^B = P_j^A \Rightarrow$ $P_o^a \cap P_l^B \neq \varnothing,\ P_o^a \cap P_m^B \neq \varnothing,\ P_o^a \cap P_n^B = \varnothing$
Subset Rule (A $\subset$ B)	rule 3	$P_o^a \subset P_i^A,\ P_i^A \subset P_l^B \Rightarrow$ $P_o^a \cap P_l^B \neq \varnothing,\ P_o^a \cap P_m^B \neq \varnothing,\ P_o^a \cap P_n^B = \varnothing$
	rule 4	$P_o^a \cap P_i^A \neq \varnothing,\ P_o^a \cap P_j^A \neq \varnothing,\ P_i^A \subset P_l^B \Rightarrow P_o^a \cap P_l^B \neq \varnothing$
	rule 5	$P_o^a \cap P_i^A \neq \varnothing,\ P_o^a \cap P_j^A \neq \varnothing,\ P_i^A \subset P_l^B,\ P_j^A \subset P_l^B \Rightarrow$ $P_o^a \cap P_l^B \neq \varnothing,\ P_o^a \cap P_m^B = \varnothing,\ P_o^a \cap P_n^B = \varnothing$
	rule 6	$P_o^a \subset P_i^A,\ P_i^A \cap P_l^B \neq \varnothing,\ P_i^A \cap P_l^B \neq \varnothing,\ P_i^A \cap P_m^B \neq \varnothing,$ $P_i^A \subset (P_l^B \cup P_m^B) \Rightarrow$ $P_o^a \cap P_l^B \neq \varnothing,\ P_o^a \cap P_m^B \neq \varnothing,\ P_o^a \cap P_n^B = \varnothing$
Super-set Rule (A $\supset$ B)	rule 7	$P_o^a \cap P_i^A \neq \varnothing,\ P_o^a \cap P_j^A \neq \varnothing,\ P_i^A \cap P_l^B \neq \varnothing,\ P_i^A \cap P_m^B \neq \varnothing,$ $P_i^A \subset (P_l^B \cup P_m^B) \Rightarrow P_o^a \cap P_l^B \neq \varnothing,\ P_o^a \cap P_m^B \neq \varnothing,$
	rule 8	$P_l^B \subset P_i^A,\ P_o^a \subset P_j^A \Rightarrow P_o^a \cap P_l^B = \varnothing$
	rule 9	$P_l^B \subset P_i^A \wedge P_m^B \subset P_i^A \wedge P_o^a \subset P_i^A \Rightarrow P_n^B \cap P_p^a \neq \varnothing,$ $P_n^B \cap P_q^a \neq \varnothing$, where $P_l^B \neq B^-,\ P_m^B \neq B^-,\ P_i^A \neq A^-$
	rule 10	$P_o^a \cap P_i^A \neq \varnothing,\ P_o^a \cap P_j^A \neq \varnothing,\ P_l^B \subset P_i^A \Rightarrow$ $\sim (P_o^a \cap P_m^B = \varnothing \wedge P_o^a \cap P_n^B = \varnothing)$
Empty-set Rule $A \cap B = \varnothing$	rule 11	$P_o^a \subset P_i^A,\ P_i^A \cap P_l^B = \varnothing \Rightarrow P_o^a \subset P_l^B = \varnothing$
	rule 12	$P_o^a \cap P_i^A \neq \varnothing,\ P_o^a \cap P_j^A \neq \varnothing,\ P_l^B \cap P_i^A = \varnothing, P_m^B \cap P_i^A = \varnothing$ $\Rightarrow P_o^a \cap P_l^B = \varnothing$
Non-empty -set Rule $A \cap B \neq \varnothing$, $A \cap B \neq A$, $A \cap B \neq B$	rule 13	$(P_o^a \cup P_p^a) \subset P_i^A \wedge P_j^A \cap P_l^B \neq \varnothing \Rightarrow P_q^a \cap P_l^B \neq \varnothing$
	rule 14	$P_o^a \cap P_i^A \neq \varnothing,\ P_p^a \cap P_i^A \neq \varnothing,\ P_o^a \cap P_j^A \neq \varnothing,\ P_p^a \cap P_j^A \neq \varnothing,$ $P_k^A \cap P_l^B \neq \varnothing \Rightarrow P_q^a \cap P_l^B \neq \varnothing$
	rule 15	$P_o^a \cap P_i^A \neq \varnothing,\ P_o^a \cap P_j^A \neq \varnothing,\ P_i^A \cap P_l^B = \varnothing,\ P_j^A \cap P_l^B = \varnothing$ $\Rightarrow P_o^a \cap P_l^B = \varnothing$
	rule 16	$P_o^a \subset P_i^A \wedge P_i^A \cap P_l^B = \varnothing \Rightarrow P_o^a \cap P_l^B = \varnothing$

Notaion

- $^\circ$: Interior, ∂ : Boundary, $^-$: Exterior
- $P_i^A, P_j^A, P_k^A \in \{A^\circ, \partial A, A^-\}$, where $P_i^A \neq P_j^A \neq P_k^A$, $i \neq j$, $j \neq k$, $k \neq i$.
- $P_l^B, P_m^B, P_n^B \in \{B^\circ, \partial B, B^-\}$, where $P_l^B \neq P_m^B \neq P_n^B$, $l \neq m$, $m \neq n$, $n \neq l$.
- $P_o^a, P_p^a, P_q^a \in \{a^\circ, \partial a, a^-\}$, where $P_o^a \neq P_p^a \neq P_q^a$, $o \neq p$, $p \neq q$, $q \neq o$.

Table 3. Rules for Collapse-type

Collapse-Type		A=B	$A \subset B$	$A \supset B$	$A \cap B = \varnothing$	$A \cap B \neq A, B \wedge A \cap B \neq \varnothing$
type 1	a°	rule 1	rule 3 or 6	rule 8 or 9	rule 11	rule 13 or 16
	∂a	rule 1	rule 3, 6 or 7	rule 8 or 9	rule 11	rule 13 or 16
type 2	a°	rule 1	rule 3 or 6	rule 8 or 9	rule 11	rule 13 or 16
	∂a	rule 1	rule 3 or 6	rule 8 or 9	rule 11	rule 13 or 16
type 3	a°	rule 1	rule 3 or 6	rule 8 or 9	rule 11	rule 13 or 16
	∂a	rule 2	rule 4, 5 or 7	rule 10	rule 12	
type 4	a°	rule 1	rule 3 or 6	rule 8 or 9	rule 11	rule 13 or 16
	∂a	rule 1	rule 3 or 6	rule 8 or 9	rule 11	rule 13 or 16
type 5	a°	rule 1	rule 3 or 6	rule 8 or 9	rule 11	rule 13 or 16
	∂a	rule 1	rule 3 or 6	rule 8 or 9	rule 11	rule 13 or 16
type 6	a°	rule 1	rule 3 or 6	rule 8 or 9	rule 11	rule 13 or 16
	∂a	rule 2	rule 4, 5 or 7	rule 10	rule 12	rule 14 or 15
type 7	a°	rule 2	rule 4, 5 or 7	rule 10	rule 12	rule 14 or 15
	∂a	rule 1	rule 3 or 6	rule 8 or 9	rule 11	rule 13 or 16
type 8	a°	rule 2	rule 4, 5 or 7	rule 10	rule 12	rule 14 or 15
	∂a	rule 1	rule 3 or 6	rule 8 or 9	rule 11	rule 13 or 16
type 9	a°	rule 2	rule 4, 5 or 7	rule 10	rule 12	rule 14 or 15
	∂a	rule 2	rule 4, 5 or 7	rule 10	rule 12	rule 14 or 15

i. relationship between a° and B :
According to table 4, rule 1 ($P_o^a \subset P_i^A, P_l^B = P_i^A \Rightarrow P_o^a \cap P_l^B \neq \varnothing, P_o^a \cap P_m^B = \varnothing$, and $P_o^a \cap P_n^B = \varnothing$) must be applied for this case. Let $a^\circ = P_o^a$ and $A^\circ = P_i^A$, then $P_l^B = B^\circ$ since $A^\circ = B^\circ$. And P_m^B and P_n^B become ∂B and B^-, respectively. Therefore by substituting them to `rule 1`, we obtain $a^\circ \cap B^\circ \neq \varnothing, a^\circ \cap \partial B = \varnothing$, and $a^\circ \cap B^- = \varnothing$.

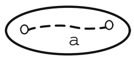
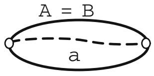
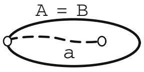
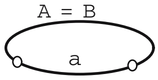
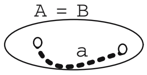
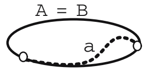
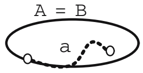
ii. relationship between ∂a and B :
By similar way, we obtain $\partial a \cap B^\circ \neq \varnothing, \partial a \cap \partial B = \varnothing$, and $\partial a \cap B^- = \varnothing$ by putting $\partial a = P_o^a$ and $A^\circ = P_i^A$ according `rule 1`.

iii. relationship between a° and B^- :
If the exterior of a intersects with the interior, boundary, and exterior of B, it is evident that $a^- \cap B^\circ \neq \varnothing, a^- \cap \partial B \neq \varnothing$, and $a^- \cap B^- \neq \varnothing$, and $R(a^-, B) = (1, 1, 1)$. Thus, we exclude this case from table 2 for this reason.

Consequently we conclude that the 9-IM matrix between a and B is as follows,

$$R(a, B) = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

Table 4. Equal-set Rule Example

<table>
<tr><th colspan="2">R(A,a)
Collapse-Type</th><th rowspan="2">R(A,B)
$(A = B)$</th><th rowspan="2">Example</th><th colspan="2">R(a,B)</th></tr>
<tr><th></th><th></th><th>Derivation Process</th><th>Consistent PL</th></tr>
<tr><td rowspan="2">type 1</td><td>a°</td><td>rule 1</td><td rowspan="2">A = B

$a^\circ \subset A^\circ \wedge A^\circ = B^\circ$
$\partial a \subset A^\circ \wedge A^\circ = B^\circ$</td><td rowspan="2">$\begin{array}{c|ccc} a \backslash B & \circ & \partial & - \\ \hline \circ & 1 & 0 & 0 \\ \partial & 1 & 0 & 0 \\ - & 1 & 1 & 1 \end{array}$</td><td rowspan="2">none</td></tr>
<tr><td>∂a</td><td>rule 1</td></tr>
<tr><td rowspan="2">type 2</td><td>a°</td><td>rule 1</td><td rowspan="2">A = B

$a^\circ \subset A^\circ \wedge A^\circ = B^\circ$
$\partial a \subset A^\circ \wedge \partial A = \partial B$</td><td rowspan="2">$\begin{array}{c|ccc} a \backslash B & \circ & \partial & - \\ \hline \circ & 1 & 0 & 0 \\ \partial & 0 & 1 & 0 \\ - & 1 & 1 & 1 \end{array}$</td><td rowspan="2">$PL_8$</td></tr>
<tr><td>∂a</td><td>rule 1</td></tr>
<tr><td rowspan="2">type 3</td><td>a°</td><td>rule 1</td><td rowspan="2">A = B

$a^\circ \subset A^\circ \wedge A^\circ = B^\circ$
$\partial a \subset A^\circ \cup \partial A \wedge \partial A = \partial B$</td><td rowspan="2">$\begin{array}{c|ccc} a \backslash B & \circ & \partial & - \\ \hline \circ & 1 & 0 & 0 \\ \partial & 1 & 1 & 0 \\ - & 1 & 1 & 1 \end{array}$</td><td rowspan="2">$PL_{10}$</td></tr>
<tr><td>∂a</td><td>rule 2</td></tr>
<tr><td rowspan="2">type 5</td><td>a°</td><td>rule 1</td><td rowspan="2">A = B

$a^\circ \subset \partial A \wedge \partial A = \partial B^\circ$
$\partial a \subset \partial A \wedge \partial A = \partial B$</td><td rowspan="2">$\begin{array}{c|ccc} a \backslash B & \circ & \partial & - \\ \hline \circ & 0 & 1 & 0 \\ \partial & 0 & 1 & 0 \\ - & 1 & 1 & 1 \end{array}$</td><td rowspan="2">$PL_4$</td></tr>
<tr><td>∂a</td><td>rule 1</td></tr>
<tr><td rowspan="2">type 7</td><td>a°</td><td>rule 2</td><td rowspan="2">A = B

$a^\circ \subset A^\circ \cup \partial A \wedge \partial A = \partial B$
$\partial a \subset A^\circ \wedge A^\circ = B^\circ$</td><td rowspan="2">$\begin{array}{c|ccc} a \backslash B & \circ & \partial & - \\ \hline \circ & 1 & 1 & 0 \\ \partial & 1 & 0 & 0 \\ - & 1 & 1 & 1 \end{array}$</td><td rowspan="2">$PL_{12}$</td></tr>
<tr><td>∂a</td><td>rule 1</td></tr>
<tr><td rowspan="2">type 8</td><td>a°</td><td>rule 2</td><td rowspan="2">A = B

$a^\circ \subset A^\circ \cup \partial A \wedge A^\circ = B^\circ$
$\partial a \subset \partial A \wedge \partial A = \partial B$</td><td rowspan="2">$\begin{array}{c|ccc} a \backslash B & \circ & \partial & - \\ \hline \circ & 1 & 1 & 0 \\ \partial & 0 & 1 & 0 \\ - & 1 & 1 & 1 \end{array}$</td><td rowspan="2">$PL_{11}$</td></tr>
<tr><td>∂a</td><td>rule 1</td></tr>
<tr><td rowspan="2">type 9</td><td>a°</td><td>rule 2</td><td rowspan="2">A = B

$a^\circ \subset A^\circ \cup \partial A \wedge A^\circ = B^\circ$
$\partial a \subset A^\circ \cup \partial A \wedge \partial A = \partial B$</td><td rowspan="2">$\begin{array}{c|ccc} a \backslash B & \circ & \partial & - \\ \hline \circ & 1 & 1 & 0 \\ \partial & 1 & 1 & 0 \\ - & 1 & 1 & 1 \end{array}$</td><td rowspan="2">$PL_{13}$</td></tr>
<tr><td>∂a</td><td>rule 2</td></tr>
</table>

Notation.

R(A,B) : relation of A and B.

PL : Polygon Line relationship in[2].

rule 1 : $P_o^a \subset P_i^A,\ P_l^B = P_i^A \ \Rightarrow \ P_o^a \cap P_l^B \neq \varnothing,\ P_o^a \cap P_m^B = \varnothing,\ P_o^a \cap P_n^B = \varnothing$

rule 2 : $P_o^a \cap P_i^A \neq \varnothing,\ P_o^a \cap P_j^A \neq \varnothing,\ P_l^B = P_i^A,\ P_m^B = P_j^A \ \Rightarrow$
$P_o^a \cap P_l^B \neq \varnothing,\ P_o^a \cap P_m^B \neq \varnothing,\ P_o^a \cap P_n^B = \varnothing$

when $A = B$ and the collapse type is type 1 according to table 2, 3, and 4. For other cases, we can derive the matrix by similar way and assess the topological consistency between the original and derived databases.

5 Conclusion

Topological changes take place during the map generalization process from a spatial database of large scale to small scaled database. These changes often result in an inconsistency between the original and derived databases. In order to control the quality of derived databases, the topological consistency must be maintained. For this, we proposed a set of rules, which describes the consistent correspondence between the topologies in the original and derived databases for the collapse operation of generalization.

These rules are based on the classification of the collapse operation by using boundary and interior topology between the original and derived spatial objects. Therefore describing the possible topological changes from the original database. Also through such findings, one can detect inconsistent topological changes in derived databases.

In this study, we have dealt with only the collapse operation from polygonal object to line object and exclude the case where the geometric type of derived object is point. Due to the fact that the polygon-point topology is much simpler than polygon-line or line-line topologies, the proposed rules in this paper can be easily extended. Such is the ground work for future research in this field. Moreover, future endeavors can also include the study on topological consistency for simplification operation in addition to collapse operation.

Acknowledgements

This research was partially supported by the Program for the Training of Graduate Students in Regional Innovation which was conducted by the Ministry of Commerce, Industry and Energy of the Korean Government and by the Internet information Retrieval Research Center(IRC) in Hankuk Aviation University. IRC is a Regional Research Center of Kyounggi Province, designated by ITEP and Ministry of Commerce, Industry and Energy.

References

1. M. J. Egenhofer and H. Herring, *Categorizing Binary Topological Relations Between Regions, Lines, and Points in Geographic Databases*, Technical Report, Department of Surveying Engineering, University of Maine, 1990.
2. M. J. Egenhofer, *Point-Set Topological Spatial Relations*, International Journal of Geographical Information Systems 5(2):161-174, 1991.
3. M. J. Egenhofer and K. K. Al-Taha, *Reasoning about Gradual Changes of Topological Relationships*, Theory and Methods of Spatio-Temporal Reasoning in Geographic Space, LNCS, VOL. 639, Springer-Verlag, 196-219, 1992.

4. M. J. Egenhofer, and J. sharma, *Assessing the Consistency of Complete and Incomplete Topological Information*, Geographical Systems, 1(1):47-68, 1993.
5. E. Clementini, J. Sharma, and M. J. Egenhofer, *Modeling Topological Spatial Relations : Strategies for Query Processing*, Computer and Graphics 18(6):815-822, 1994.
6. M. Egenhofer, *Evaluating Inconsistencies Among multiple Representations*, 6th international Symposium on Spatial Data Handling, 902-920, 1994.
7. M. J. Egenhofer, E. Clementini and P. Felice, *Topological relations between regions with holes*, International Journal of Geographical Information Systems 8(2):129-144, 1994.
8. M. J. Egenhofer, *Deriving the Composition of Binary Topological Relations*, Journal of Visual Languages and Computing, 5(2):133-149, 1994.
9. J. Sharma, D. M. Flewelling, and M. J. Egenhofer, *A Qualitative Spatial Reasoner*, 6th International Symposium on Spatial Data Handling, 665-681, 1994.
10. Muller J. C., Lagrange J. P. and Weibel R., *Data and Knowledge Modelling for Generalization in GIS and Generalization*, Taylor & Francis Inc., 73-90, 1995.
11. A.I. Abdelmoty and B. A. El-Geresy, *A General Method for Spatial Reasoning in Spatial Databases*, CIKM, 312-317, 1995.
12. M. J. Egenhofer, *Consistency Revisited*, GeoInformatica, 1(4):323-325, 1997.
13. N. Tryfona and M. J. Egenhofer, *Consistency among Parts and Aggregates: A Computational Model*, Transactions in GIS, 1(3):189-206, 1997.
14. A. I. Abdelmoty and C. B. Jones, *Toward of Consistency Maintenance of Spatial Database*, CIKM, 293-300, 1997.
15. J. A. C. Paiva, *Topological Consistency in Geographic Databases With Multiple Representations*, Ph. D. Thesis, University of Maine, 1998, http://library.umaine.edu/theses/pdf/paiva.pdf.
16. H. Kang, S. Do, and Ki-Joune Li, *Model-Oriented Generalization Rules*, Proc.(in CD) ESRI Conf. San Diego, USA, July, 2001.
17. H. Kang, T. Kim, and K. Li, *Topological Consistency for Collapse Operation in Multi-scale Databases*, 1st Workshop on Conceptual Modeling for Geographic Information Systems in Conjunction with ER2004, Lecture Notes in Computer Science 3289, Springer-Verlag, 91-102, 2004.